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B.Sc Mathematics
Part III

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The centre of a Group:

Def The set Z of all self-conjugate elements of a Group G is called the Centre of G. Symbolically

$$Z = \{z \in G : zx = xz \ \forall x \in G\}$$

Th 1 The centre Z of a group G is a normal subgroup of G.

Sol^n We have $Z = \{z \in G : zx = xz \ \forall x \in G\}$

First we ~~have~~ shall prove that Z is a subgroup of G.

Let $z_1, z_2 \in Z$. Then $z_1 x = x z_1$ and $z_2 x = x z_2$ for all $x \in G$.

We have, $z_2 x = x z_2 \ \forall x \in G$

$\Rightarrow z_2^{-1}(z_2 x) z_2^{-1} = z_2^{-1}(x z_2) z_2^{-1}$

$\Rightarrow x z_2^{-1} = z_2^{-1} x \ \forall x \in G$

$\Rightarrow z_2^{-1} \in Z$.

Now ~~since~~ $(z_1 z_2^{-1}) x = z_1 (z_2^{-1} x)$

$= z_1 (x z_2^{-1}) = (z_1 x) z_2^{-1} = (x z_1) z_2^{-1}$

$= x (z_1 z_2^{-1})$

$\therefore z_1 z_2^{-1} \in Z$

Thus $z_1, z_2 \in Z \Rightarrow z_1 z_2^{-1} \in Z$

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$\therefore$ Z is a Subgroup of G.

Now we shall show that Z is a normal Subgroup of G. Let $x \in G$ and $z \in Z$
Then

$$xzx^{-1} = (xz)x^{-1} = (zx)x^{-1} = z \in Z$$

Thus $x \in G,\ z \in Z \Rightarrow xzx^{-1} \in Z$

$\therefore$ Z is a normal subgroup of G.